Turbulent diffusion in Lake Huron

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The diffusion of small floating objects and of fluorescent dye has been studied at Douglas Point, Lake Huron. The dispersal of floating objects was complicated by surface *confluences*, slicks and windrows, which under certain circumstances could completely reverse the diffusion process. In the absence of such disturbing effects, however, the dispersal of floating objects exhibited an *increase* in rate of growth with the size of the diffusing cloud, characteristic of relative turbulent diffusion. A similar conclusion holds with regard to the diffusion of dye perpendicular to the mean current. Along the direction of the current the diffusion of dye was *accelerated* by the changes of current velocity with depth, much as in a pipe or channel. In the vertical, diffusion was very slow, the effective diffusivity being barely 10 times the molecular constant. Even in the horizontal the effective diffusivities describing relative diffusion were much less than values typical of the lower atmosphere or of the ocean (only of order 10^3 times the molecular diffusivity). A study of the *meandering* of the dye plume showed that this was a more important agency than relative diffusion in dispersing the dye over a large area.

1. Introduction

In the summer of 1962 an intensive field research programme was carried out by the Great Lakes Institute, University of Toronto, with the object of studying turbulent diffusion, among other phenomena, in Lake Huron at Douglas Point, where the first nuclear reactor in Canada for commercial operation is now under contruction. Some preliminary work had been done in the summer of 1961 and some further studies are proposed for 1963.

The details of the work have been described in internal reports of the Great Lakes Institute. In the present paper an account is given of the more general aspects of the results concerning turbulent diffusion. Diffusion experiments were carried out in general with floating objects and with fluorescent dye.

The dispersal of floating objects on some water surface, ocean or lake, is a relatively easily observed manifestation of turbulent diffusion. Observations on *pairs* of floating objects have been reported by Richardson & Stommel (1948), for example, in an attempt to provide data on *relative* diffusion. At Douglas Point the dispersal of *groups* of floating objects was observed in a number of experiments. The growth of group size with time or distance drifted provided data theoretically equivalent to those obtained by observations in pairs of objects

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with less statistical scatter. Also some information could in this manner be obtained on the probability distribution of objects in a diffusing group.

It was observed quite early in the course of this work that the diffusion of floating objects may show a peculiar behaviour owing to the fact that such objects take a 2-dimensional section of a 3-dimensional flow field. In particular, there exist lines or regions of *confluence*, into which surface water converges from several directions and then sinks. When a diffusing group, initially dispersing regularly, encounters a strong confluence, the surface water turns downward, disappearing from between the floating objects and one may be faced with the extraordinary phenomenon of a complete reversal of the diffusion process, a re-agglomeration of the dispersing objects.

A strong confluence which remains stationary for a period could be regarded as something akin to a spatial boundary. There are, however, moving confluences associated with thermocline waves and also temporary confluences set up by the wind ('windrows'). There is no alternative to regarding these as part of the diffusing field, even though such confluences collect and organize floating objects instead of dispersing them.

Fortunately, confluences were not always present and in about half of the experiments the dispersal of floating objects was 'regular', i.e. the group grew fairly regularly, with the rate of growth increasing with group size. In such cases it was legitimate to regard the field as homogeneous both 'longitudinally' (along the direction of the mean flow, the current) and 'laterally' (perpendicular to it) so that Batchelor's (1952*a*) analysis of relative diffusion could be made the basis of interpreting the results.

The analogue in water of a smoke plume is a dye plume which forms when dye is introduced at a constant rate into a steady current. When the source is situated at the surface of the water it is quite easy to sample the plume from a small boat and one may obtain data on *relative* diffusion much more readily than in the atmosphere. At the same time the plume may be photographed from the air a number of times in succession in order to study its bodily movement or *meandering*. This experiment was also carried out a number of times at Douglas Point in the summer of 1962. Aerial photographs of diffusing dye patches (rather than plumes) were taken in the summer of 1961.

The behaviour of dye plumes and patches was found to depend strongly on the prevailing current patterns. In a regular, well-established current a well-defined and relatively narrow plume formed which could be studied easily. This was most often the case in a moderately steady wind. Changing winds produced erratic, shifting currents varying in a complex manner along the vertical and this led to corresponding complications in the dispersion of dye. It goes without saying that meaningful quantititive data could only be collected in regular plumes.

Numerical results are given in detail below. One general observation that may be made here is that diffusion rates in the lake are considerably lower than those typical either of the lower atmosphere or the ocean. The appearance of a dye-plume as photographed from the air reminds one of the condensation trails of a high-flying aeroplane rather than of a smoke plume.

Most of the data collected concern the dispersal of dye in a lateral direction.

Diffusion in a homogeneous field is a satisfactory theoretical model for this. Along the vertical, diffusion is very much slower and also considerably more complicated. Owing to lack of time only scant data could be collected on vertical diffusion. On diffusion in the longitudinal direction experimental data are provided by the aerial photographs of diffusing dye patches. Longitudinal diffusion of dye is again not a simple process as it is bound up with vertical variations of current velocity, much as in longitudinal diffusion in a pipe, analysed by Taylor (1954). Some rough calculations as well as the experimental evidence indicates that longitudinal diffusion in a steady lake-current is considerably faster than in the lateral direction.

2. Experimental methods

(a) *Dispersal of floating objects*

Two kinds of floating objects were used in most of the experiments: small cylindrical jars and ordinary foolscap-size mimeograph paper. The jars were 5 cm in diameter, 8 cm high and their lids were painted fluorescent yellow for ease of recognition. The jars were weighted so that they would barely float and their lids would be level with the surface (within 2 or 3 mm), in order to eliminate any wind-drag on them. The sheets of paper floated very close to the surface almost entirely flat, certainly within the top 1 cm of water. Owing to a strong velocity gradient near the surface the sheets of paper usually drifted faster than the bottles, because the latter effectively averaged the velocity in the top 8 cm of water. The difference in mean velocity was of the order of 20 %, but otherwise the two kinds of objects diffused identically: the growth of the group size with *distance* drifted was the same, within the expected statistical scatter. At the end of an experiment the bottles were retrieved by means of a landing net. Out of sixty bottles usually released, one, two or three were lost in each trial. In general, near the end of a trial the group of bottles was usually scattered over a large area and it was difficult to locate the stragglers, particularly under rough conditions. For this reason there was a tendency to underestimate the group size during the last phase of an experiment wherein bottles were used. No such difficulty was experienced with drifting papers, owing to their much greater size.

The location of a group and its size were found with the aid of two transit stations set up on the shore. On a flag signal from a boat simultaneous readings were taken at the transits. In the initial stages of diffusion it was only practicable to give one signal per group and reading, and the size of the group was then estimated from the boat by comparison with the boat's length. When the group grew large enough, two signals could be given, one each at the front and at the rear end of the group. Subsequently, a correction was applied for the mean drift between the two signals. The 'front' and the 'rear' of the group was deemed to be the first and the last object respectively along the longest diameter, which was usually nearly parallel to the direction of the mean drift. The distance between these two objects is called 'group diameter' or 'group size' herein. As a very crude measure, one may take the group size to be four standard deviations for the purpose of calculating diffusion constants: this has been done in processing the data. The shore transit stations were permanently marked and their position was accurately located on a large-scale map by a survey carried out for this purpose. At the end of a trial the transit readings were transferred to the map and mean drift and group size determined.

The accuracy of locating individual points by transits is quite high (in all the experiments within 0·5 m or better) but some of this was lost owing to the practical difficulty of holding the signal flag exactly above a given drifting object. Also, it took several seconds to make a transit reading while the boat typically drifted 1 m in 5 sec. Therefore the probable error of locating the diffusing objects may be estimated at 3–5 m, the higher figure to hold for rough conditions. It was for this reason that whenever a group was smaller than about 15 m, its size was estimated by eye from the boat, rather than by two transit signals.

On one occasion (run no. 9) a batch of standard surface drogues were released. These consisted of two metal sheets 60 cm deep by 90 cm long, assembled at 90°, so as to offer a high drag, connected to a surface float. These drogues effectively averaged the velocity between the depths of approximately 25 and 85 cm and were therefore slower than either bottles or papers, although the difference in speed against the bottles was no more than about 10%. The advantage of surface drogues was that being larger objects they were more easily traced individually: a marker flag was mounted on their floats and it was possible to aim the transits at these flags. In order to find the simultaneous position of the whole group it was necessary to interpolate between the readings for each drogue. Subsequently mean drift and standard deviations could be calculated. The computations for this run were carried out on an electronic computer at Assumption University, Windsor.

In this last-mentioned drogue run, data on particle distribution within the group could also be collected, although owing to the relatively small number of drogues (11) the statistical scatter was great. In other experiments only qualitative estimates of distribution could be made by eye.

(*b*) *Diffusion of dye*

In the summer of 1962 all dye diffusion experiments at Douglas Point were conducted with the aid of a simple constant-head syphon apparatus (figure 1) which discharged dye at a constant rate into the lake. The apparatus was usually located on a fixed instrument tower 800 m from shore with the nozzle a short distance above water level, so that the arrangement could be described as a 'point source', emitting dye at the water surface. The rate of discharge of the apparatus depended on the nozzle and on the constant head Δh, and was usually around 1·5 gal./h or approximately 2 g/sec. A photograph of the dye source on the instrument tower is shown in figure 2.

The fluorescent dye used was Rhodamine B (I.C.I. Ltd) and this was mixed with water and alcohol so as to give a liquid of specific gravity 1·0 containing the dye in a concentration of 10^{-1}. In practice this was achieved by dissolving 5 lb. of solid dye in a mixture of 2 gal. of alcohol and some water, adding enough water to give 5 gal. of solute. Pritchard & Carpenter (1960) describe some of the practical advantages of this dye.

In a steady current of approximately constant speed the dye source produced a slender, continuous plume extending a mile or two downstream. This plume was sampled from a 14 ft. boat moving roughly at right angles to the plume and reasonably fast (at 1–1·5 m/sec). The sample was drawn continuously at a level fixed for any one crossing and passed through a constant-flow door Turner fluorometer equipped with a strip-chart recorder. In this manner a continuous graph of concentration versus time was obtained. The boat's speed was kept constant during a crossing and measured with the aid of a propeller-type current meter (Ott meter). Using the measured value of boat speed, the time co-ordinate on the concentration graph could be converted to a distance co-ordinate.

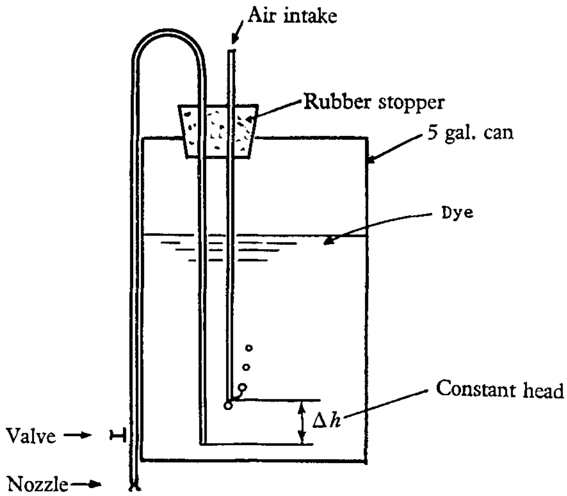

FIGURE 1. Constant head syphon apparatus, used as the dye source.

In order to determine the distance of the sample section from the source two marker buoys were usually anchored near the outer edges of the plume. These were surveyed from two transit stations set up on shore and their position determined accurately. The main advantage of the marker buoys was that they could be used to avoid drift of the boat with the current. A number of profiles at the same cross-section could in this manner be obtained. On occasions when this was not considered necessary the position of the boat itself was surveyed from the shore on a flag signal.

With the flow rates of dye chosen for the experiments, the concentration was quite high for some 500 m from the source and neutral density filters had to be employed in the fluorometer, in order to establish the value of the peak concentration. The amount of secondary (fluorescent) light leaving the sample had to be reduced on occasion by a factor of as high as 100. The filters used were ordinary Kodak ND 2·00, 1·00 and 0·60 gelatine filters. At larger distances from the source the filters were not required, but the least sensitive scale of the instrument had still to be used as far downstream as 1 mile. Owing to lack of time the plume could not be followed far enough to utilize the very high sensitivity of the fluorometer. In consequence, no problems were encountered in connexion with background fluorescence, because on the least sensitive scale lake water was little different from a blank sample.

The pump circulating the sample through the instrument was mounted on a boom which could be lowered to about 3 m below the surface, in order to obtain cross-sections at different levels. It would have been a simple matter to extend the sampling depth, but lack of time and of suitable weather prevented the full exploitation of the experimental facility. Since the opportunities for diffusion experiments were limited, efforts were at first concentrated on obtaining data on the lateral growth of the plume. For this purpose concentration profiles were taken at a fixed level, as close to the surface as possible but without entraining air in the pump intake. Air bubbles had to be eliminated because they caused the fluorescence reading to jump sharply. Owing to the rocking of the boat, the sampling intake, when firmly fixed to the boat, had to be at least 60 cm below the surface, as measured on calm water.

At the 60 cm level a number of concentration profiles were collected at varying distances from the source. In these, the edge of the plume was usually marked by a sharp rise of concentration and so a plume width could be easily defined. A quarter of this total width, from one edge to the other, was arbitrarily designated the lateral standard deviation, S_y.

The individual instantaneous concentration profiles were very irregular and an attempt was made to obtain 'average' relative plume profiles by taking a number of profiles at the same distance from the source: this is why anchored marker buoys were set out, as already recorded. The distance between two buoys (determined by survey) provided a check on the accuracy of distance measurement across the plume by current meter from the boat. The results were reasonably close but there was a general tendency to overestimate distances by up to 10 % when using the current meter.

Another reason for probable overestimation of plume size is that the boat's path was only very approximately perpendicular to the plume axis, as well as it could be judged by eye. However, in view of the crudeness of the definition of S_y these are relatively minor causes of error.

The discharge rate of the dye sources employed was directly measured and so, with the aid of current speed and lateral plume size, estimates of vertical plume size could be made. In addition some direct concentration measurements obtained at depths up to 3 m are available although, as remarked before, this information is rather inadequate. Current speeds were determined by timing the travel of the front edge of the plume over a long distance, beginning from the time when the flow of the dye at the source was started. This is likely to be an overestimate of the actual transport velocity of dye at any given cross-section. Steady lake currents were mostly wind-driven and the maximum velocity occurred at the surface. The velocity gradient near the surface is quite sharp under such conditions so that the average transport velocity at a given section could be considerably less than the surface velocity. Now, owing to vertical diffusion the speed of the visible front edge of a plume is likely to be less than the surface velocity, but it may still be considerably above the average transport velocity at a given section. It was for this reason difficult to assess the balance of dye accurately.

In order to establish the connexion between the dial reading on the fluorometer and dye concentration the instrument was calibrated by means of a continuous-flow arrangement in the *ad hoc* laboratory on the site. The accuracy of the result should be within ± 5 %.

In addition to the fluorometer readings, aerial photographs have been obtained of diffusing dye patches (1961) and dye plumes (1962) with the kind co-operation of the R.C.A.F. The connexion between visibility of a plume or patch and dye concentration is rather uncertain so that the photographs are mainly of qualitative value as far as diffusion of the dye goes. The main use of the 1962 photographs was that they provided data on *meandering*. At the same time, they provided some check on the reliability of lateral diffusion results obtained by the fluorometer. The 1961 photographs have yielded information on the ratio of longitudinal to lateral diffusion which is also important in connecting the dye-diffusion data to results on the diffusion of *floating* objects.

The aerial photography experiments took place on 15 August and 20 August 1962. On the first occasion three plumes were photographed, on the second occa sion two. Unfortunately, the current on the first day was highly variable and only a fraction of the photographs taken were usable. On the second day conditions were quite favourable and a good series of pictures were obtained, particularly on the plume started further out from shore (approximately 800 m).

The photographs of this last-named plume were analysed with the aid of a sheet of transparent plastic, grid-lined at appropriate intervals, which was placed over the photographs. The position of the centre of a plume (as estimated by eye) was marked at a number of fixed distances in all photographs analysed. The mean lateral position of the plume in a group of readings at a fixed distance from the source was obtained using a desk calculator, and so was the root-mean-square difference from the mean. The maximum distance for which a significant number of readings (12, to wit) were available was 2100 m, but data were more plentiful in the distance range 0–1000 m.

One difficulty quickly became apparent: the variance of meandering increased with the time of sampling in a very pronounced way. This could have been anticipated: the same difficulty bedevils fixed-point diffusion measurements in the atmosphere. In *relative* diffusion the size of the group sets a clear limit to the effect on eddy size and no difficulty arises with sampling times. Since fixed-point or absolute diffusion may be regarded as the sum of relative diffusion and meandering, it follows that all the sampling-time influence is handed down through meandering or that this particular trouble is likely to be present in meandering to an accentuated degree.

For this reason meaningful comparisons could only be made if the variance of meandering was calculated for a set sampling time. Aerial photographs were taken at approximately 15 min intervals so that to obtain 12 readings (a bare minimum for calculating average position and root-mean-square deviation) a sampling time of almost 3 h was necessary. To obtain $\frac{3}{4}$ and $1\frac{1}{2}$ h averages standard deviations of 3 and 6 readings respectively were calculated for 8 and 4 such periods and the arithmetic average of these taken.

3. Current patterns

For various practical reasons the number of experiments in which dye was used during the summer of 1962 was not very great. Yet, one could distinguish between 'regular' dye plumes which formed when a steady and uniform current existed, and 'irregular' ones associated with weak and rapidly changing currents. An aerial photograph of an 'irregular' situation is shown in figure 3. Three plumes started in fairly close proximity all moved in different directions, all at a very slow speed. Between the outer two plumes there was obviously a confluence so that this case is characterized by gross spatial inhomogeneity. On another occasion, the current was very slow and its direction was changing within wide limits at random, so that in place of a plume there was an irregularly shaped pool of dye near the source. The situation was perhaps similar to what one may observe in the atmosphere on a calm day when non-buoyant smoke is discharged near ground level. As far as could be judged by this summer's experiences such freak situations are relatively rare: mostly there is a well-established current which retains its direction for several hours.

In addition to the experiments with dye and with floating objects some observations were carried out with the aid of depth-drogues and with a 'grid' of surface drogues to elucidate current patterns. The conclusions may be summarized under three headings: variation of current with (*a*) time, (*b*) horizontal location, and (*c*) depth. Greater detail is available in the *Annual Report* on these activities (Csanady & Ellenton 1962).

The variation of surface current with time was determined by local winds. Except on near-calm days the surface current was nearly parallel to the wind. On a change of wind the direction of the surface current changed within minutes, although it took a longer time for the momentum to diffuse downward. Thus on days with moderately strong, steady winds there was a correspondingly steady and uniform current the direction of which was as nearly parallel to the wind as could be, taking into account the presence of the shores.

Some horizontal variations of the current were imposed by the presence of the shores. Douglas Point juts out into the lake and causes some crowding of the streamlines. Apart from this, the frictional influence of the shores extends 250–800 m into the lake depending on bottom topography. Neither effect is likely to have influenced the results on diffusion appreciably.

Current variations with depth mainly exist following a change of wind and under such conditions the spreading of dye becomes very irregular. However, even regular currents were not uniform with depth down to the thermocline: there is a 'velocity profile', a change of mean velocity magnitude which is quite considerable and there is sometimes also a slow change of direction. At least during the daytime, currents are mainly wind-driven and then the top layers are faster than those lying below, but the direction of the flow remains constant with depth. In the region where the diffusion experiments were carried out the depth of the lake varies from 5 to 20 m. The average depth of the thermocline in July and August was approximately 15 m. The development of the thermocline was a general process during the month of June.

4. Slicks and windrows

As mentioned in the introduction, surface confluences act as 'organizing' agencies in the dispersal of floating objects in so far as they can undo the work of random eddies which disperse a group. One may distinguish *permanent* and *transitory* confluences. The former arise, for example, when the surface current at a large distance from the shore has a shoreward component. Somewhere before reaching the shore this flow must stop, the surface waters dive downward (and flow out again, one would surmise). The result is a 'slick' roughly parallel to the shore where floating debris collects. A slick is permanent as long as the current pattern in the area is more or less unchanged, but it disappears, for example, on a change of wind. Objects collected in a slick often travel along the direction

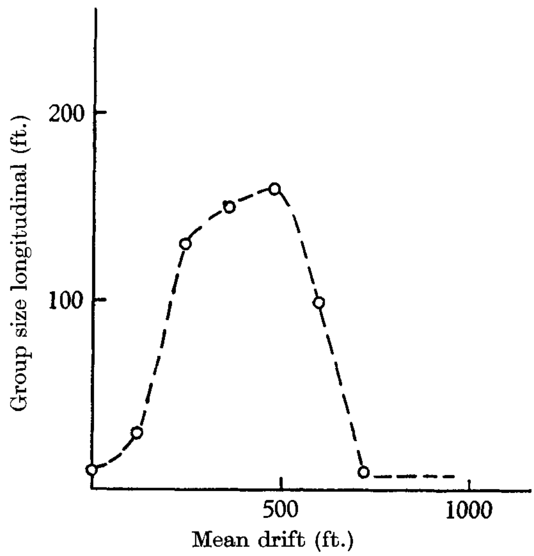

FIGURE 4. Complete reversal of diffusion by a permanent slick. Run no. 2, 28 May, dispersal of bottles.

of the slick at a considerable speed and even diffuse to some extent longitudinally, but certainly not laterally, the width of the slick being a constant 2–3 m, its length possibly many miles. Surface water in a slick is stagnant and hence may heat up a degree or two above the temperature of the surrounding water. Also it separates two different zones of surface water which often have different temperatures. Slicks are particularly prevalent before the development of the thermocline (Csanady, Ellenton & Deane 1962).

One kind of transitory, travelling confluence was observed to be associated with alternating patches of smooth and ruffled (capillary waves) surface. This could have been caused by travelling internal waves since the speed of advance of these patches was of the correct order of magnitude for internal waves. However, no direct evidence of internal waves could be found on such occasions and if there were such waves they were less than 0·5 m in amplitude.

Another kind of transitory confluence was found to be set up by moderately strong winds, in a direction parallel to the wind and with a characteristic spacing between the confluences or 'windrows' of something like 2–3 m. Langmuir (1938) has studied windrows on Lake George, N.Y. and has found that they are the result of a number of helical vortices with their axes parallel to the wind. The characteristic spacing of the windrows apparently depends on the depth to which the helical vortices penetrate, which seems to be the depth of the thermocline if it exists, of the lake otherwise. Langmuir also conjectured that these helical vortices are the main agents in causing a continuous mixing of the epilimnion waters, hence they are important in maintaining a sharp thermocline. Why the wind should set up cross-wise motions is unknown, but a reasonable hypothesis seems to be that the cooling of the surface layers by the wind provides the 'heat engine' which drives the vortices, much as the heating of the 'risers' does in a natural circulation boiler.

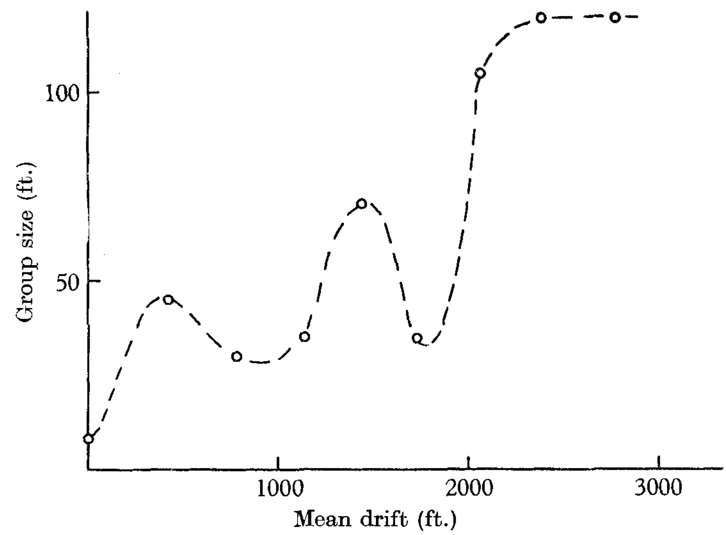

FIGURE 5. Typical case of diffusion disturbed by temporary slicks. Run no. 6, 21 June: dispersal of bottles.

Reversal or suppression of turbulent diffusion by surface confluences is illustrated by experimental data in figures 4, 5 and 6. General information on the test runs is given below in table 1. During run no. 2 (figure 4) the diffusing objects arrived at a permanent, shore-parallel slick and remained in it. During run no. 6 (figure 5) they encountered a number of transitory slicks, recognizable otherwise by their characteristic smooth surface and collected seagull feathers, etc. Following the groups from the boat the fluctuations in size were even more obvious than from the relatively crude and widely spaced data in the figure.

The experiment in which the dispersal of eleven drogues was observed by following each drogue individually deserves special treatment here. Although the group was small, the standard deviations could be calculated numerically, instead of being arbitrarily defined as $\frac{1}{4}$ group size. For an 'appointed' time the position of each drogue was calculated on the computer. It took approximately 7 min to survey the position of all drogues. Choosing the 'appointed' time near the middle of this interval it was necessary to calculate the displacement of the drogues during 0–4 min between the appointed time and the nearest reading, by calculating each drogue's velocity from successive positions. Having thus determined to a good approximation the simultaneous position of the entire group, the mean x and y co-ordinates could be found (the x-axis was the line connecting the transit stations) and also standard deviations S_x and S_y from this mean. Subsequently, standard deviations along the principal axes, S_ξ and S_η, could be calculated. Principal axes are those for which the mean product $\overline{\xi\eta}$ vanishes, the co-ordinates being measured from the centre of gravity of the group.

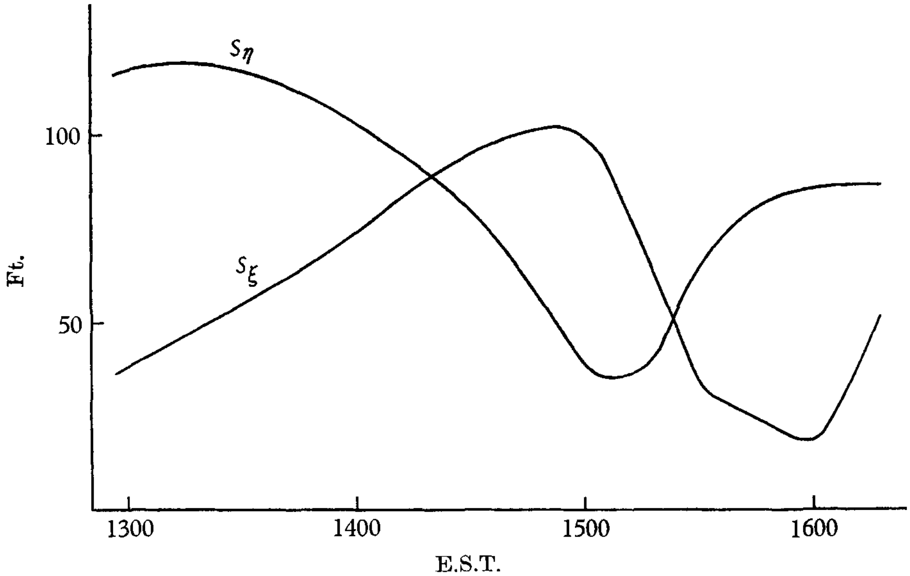

FIGURE 6. Dispersal of drogues. Run no. 9, 10 July. Standard deviations referred to principal axes versus time.

In the course of these calculations it could be verified that the distance between extreme drogues along x or y was indeed very nearly 4 standard deviations (usually within 10 %), thus justifying to some extent the procedure adopted in dealing with bottles and papers. The time variation of the standard deviations about the principal axes, S_ξ and S_η, is shown in figure 6, from which it is evident that the diffusion was far from regular. There was, in fact, no diffusion at all; the drogues were dispersed over approximately the same surface area at the end of the experiment as at the beginning of it and at certain times in between they were quite concentrated in one direction or another, and the presence of windrows was evident. No diffusion constants could be derived from this experiment. The direction of the principal axis ξ varied between $-41°$ and $+38°$, measured from the fixed x-axis.

The main use of this experiment was that it provided data on the distribution of objects in a diffusing group. Expressing the deviation from the mean position in the x and y directions in terms of the respective calculated standard deviations and summing the readings for the last 5 appointed times (there were a total of 14 of these) a histogram of the frequency distribution could be obtained, which is shown in figure 7. A Gaussian curve of the same total area is also shown in the figure, and it is obvious that the distribution is nearly Gaussian as well as can be judged from such crude data. A total of 100 individual readings went into the

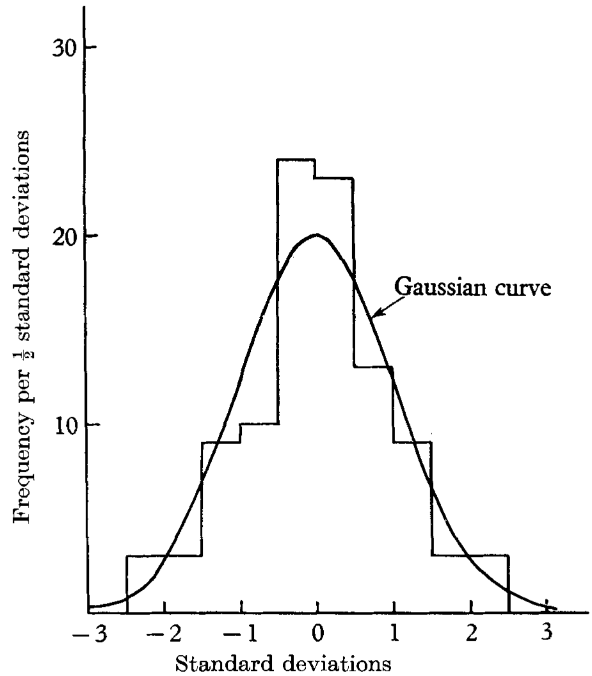

FIGURE 7. Distribution of drogues in a diffusing group. Run no. 9, 10 July.

construction of the histogram, but successive group distributions were not uncorrelated. In the initial stages of this experiment the distribution was less regular: the chance distribution in which the drogues were released disappeared only very slowly. One may sum this up in the tentative conclusion that the distribution tended to a Gaussian one, except for the organizing influence of windrows, which was evident in any individual observation, but disappeared on averaging over a longer period.

5. 'Simple' relative turbulent diffusion

The turbulent dispersal of a 'cloud' of floating objects is a problem in relative diffusion. A theoretical treatment of this problem, valid for the ideal case of a uniform, homogeneous field and steady mean velocity has been given by Batchelor (1952*a*). Horizontal diffusion of floating objects on the surface of a lake in a steady and uniform current (in the absence of permanent slicks or regularly encountered transitory confluences) should be well described by the theory. Similarly, horizontal diffusion of dye in a direction *perpendicular* to the current should at least approximately follow the same laws, except possibly for the effects of changing turbulent velocities with depth, which may influence the results as the dye spreads downward. Ignoring this complication, these two phenomena, for which Batchelor's theory is a satisfactory model, shall be termed 'simple' relative diffusion.

With the aid of dimensional arguments and adopting plausible ergodic hypotheses Batchelor finds three regimes of relative diffusion. If the initial size of the cloud is small, these regimes are characterized by the following rates of growth:

$$dS_x^2/dt = c_1 t(\epsilon S_0)^{\frac{2}{3}} \quad \text{(initial, small } t\text{);} \tag{1a}$$

$$dS_x^2/dt = c_2 \epsilon (t-t_1)^2 \quad \text{(intermediate),} \tag{1b}$$

with $t_1 = c_3 S_0^{\frac{2}{3}} \epsilon^{-\frac{1}{3}}$;

$$dS_x^2/dt = c_4 uL \quad \text{(asymptotic).} \tag{1c}$$

Here c_1 to c_4 are constants of order unity, S_x is the standard deviation measured along x, t is time, S_0 is the initial standard deviation of the cloud, ϵ is the rate of energy dissipation per unit mass through turbulence, for which the additional relationship may be noted

$$\epsilon = u^3/L, \tag{2}$$

with u the root-mean-square turbulent velocity along the x_1-axis laid along the main support and L the 'scale' of turbulence. By analogy with molecular diffusion an equivalent diffusivity may be introduced by the relationship

$$K_x = \tfrac{1}{2} dS_x^2/dt. \tag{3}$$

It is easy to see from equations (1) that in the final phase the diffusivity is constant, while in the 'intermediate' phase it grows as the $\frac{4}{3}$ power of cloud size (this may be shown on integrating equation (1*b*)). The latter is Richardson's (1926) well-known law of relative diffusion, the point of which is that relative diffusion is an accelerating process, the rate of growth increasing with the size of the cloud, at least while the supply of eddies of the requisite size lasts. When the cloud becomes large compared to the typical eddy in the diffusing field, its growth comes to resemble molecular diffusion, except for the higher order of magnitude of diffusivity.

Applied to horizontal diffusion in a lake, the above formulae may only be expected to hold as long as there is a steady current, of speed U, and of course, reasonably far from shore or from a confluence. It is convenient for the applications to introduce the 'gustiness'

$$g_x = u/U$$

and transform the time co-ordinate into distance drifted by

$$x = Ut.$$

Thus, also using equation (2), equations (1) may be rewritten as

$$dS_x^2/dx = c_1 x g_x^2 (S_0/L)^{\frac{2}{3}} \quad \text{(initial),} \tag{4a}$$

$$dS_x^2/dx = c_2 x g_x^3 x/L \quad \text{(intermediate),} \tag{4b}$$

$$dS_x^2/dx = c_4 g_x L \quad \text{(final).} \tag{4c}$$

Run no.	Date	Wind direction (°)	Wind speed (m.p.h.)	Atmos. temp. gradient (°F)	Condition of lake	Duration of test run E.S.T.	Distance from shore at point of release (m)	Depth of water at point of release (m)	Objects released
1	27 May	(315)	(4)	NA*	6 in. waves	9.15–12.00	370	10	Bottles
2	28 May	76	4·4	−1·4	6 in. swell	13.00–16.30	450	12	Bottles
3	29 May	180	9·1	−2·1	8 in. swell	8.45–10.00	360	7	Bottles
4	5 Jun	40	2·0	−0·7	6 in. swell	9.00–16.00	550	15	Bottles
5	15 Jun	250	6·9	−0·9	Calm	9.00–15.00	550	15	Bottles and papers
6	21 Jun	285	4·4	−0·5	Calm	8.30–12.05	550	15	Bottles and papers
7	21 Jun	260	6·6	−0·9	6 in. waves	13.15–15.30	1450	21	Bottles and papers
8	22 Jun	230	7·3	−0·7	28 in. waves	8.45–10.30	650	16	Papers
9	10 Jul	205	10·1	NA*	12 in. swell	12.45–16.15	850	16	Drogues

Notes. Wind directions are given in degrees. A wind blowing from the north is 0°, one blowing from the east 90°. Both wind direction and speed were measured at a meteorological tower on shore, 20 ft. above ground, the temperature 'gradient' (difference) between 20 and 80 ft. The dry adiabatic lapse rate between these levels is −0·33 °F. On days when this was not available, wind speed and direction were taken from the weather station at the site; such readings are bracketed.

* NA = Not available.

TABLE 1. General information on test runs

Here $x_1 = Ut_1$ has been ignored on the hypothesis that the initial size of the cloud, S_0, is small. On the same basis, equation (4*a*) shows that the initial rate of growth of the cloud is slow.

In equations (4*a*) to (4*c*) the important diffusion constants are gustiness g_x and scale L of the turbulence. The absence of the mean flow velocity from these formulae is notable. This suggests that the experimental results should be plotted in a form $S_x(x)$, for then the speed of mean drift, a quantity varying between wide limits, does not affect the results except through gustiness and scale. This is as with smoke plumes: the shape of the plume is more or less independent of the wind speed.

The experiments demonstrated that in 'regular' diffusion the three phases predicted by the theory are easily recognizable, in spite of the expected statistical scatter. The two diffusion constants g_x and L have been extracted from the data by estimating the slope dS_x^2/dx in the intermediate and final phases. The integrated form of equations (4*b*) and (4*c*) is also useful as a check, although the effective origin is unknown (at larger distances this is not very disturbing). The constants c_2 and c_4 have been assumed to be unity. This means that gustiness and scale may both be in error by a constant factor of order unity. The asymptotic value of diffusivity is given by

$$K_a = \tfrac{1}{2} g_x L U \tag{5}$$

and is not subject to the same errors, since this quantity is derived directly from the data.

Dates of the test runs using floating objects, weather conditions, etc., are summarized in table 1. Meteorological data were averaged for the period of the test runs.

A sample result obtained with papers is shown in figure 8, displaying 'simple' diffusion.

Numerical data extracted from the results concerning 'simple' relative diffusion only are summarized in table 2. Considering that much of the variation in the diffusion constants is due to statistical scatter, a good order-of-magnitude estimate for the asymptotic longitudinal diffusivity K would be $400\,\text{cm}^2/\text{sec}$. The scale of the turbulence L seems to be consistently of the order of 10 m while an order of magnitude estimate for gustiness would be 6–7 %.

The magnitude of the diffusion constants seems to bear no relationship to the meteorological and other data summarized in table 1. There is a perceptible negative correlation between current speed and gustiness (a swift current being associated with low gustiness) and this is one reason why the asymptotic diffusivity does not vary between wider limits.

The absence of any relationship to meteorological and other data leads one to attribute much of the variation in the diffusion constants to random statistical fluctuations. Nevertheless, there were undoubtedly 'good' days and 'bad' days for diffusion, only it has not been possible so far to associate them with external variables.

Lateral diffusion of dye is also described by equations (4) and (5), except that S_x has to be replaced by S_y and g_x by g_y, the y-axis being horizontal and perpendicular to the current. As remarked before this is an approximation in so far as variation of g_y with depth might exert an influence as the vertical size of the cloud grows (even if the direction of the current is strictly constant with depth).

The general appearance of dye plumes is illustrated in figures 9, 10 and 11. The close-up photograph in figure 9 is intended to show the sharpness of the edge of the plume near the source. It also demonstrates how the 'marked fluid' is

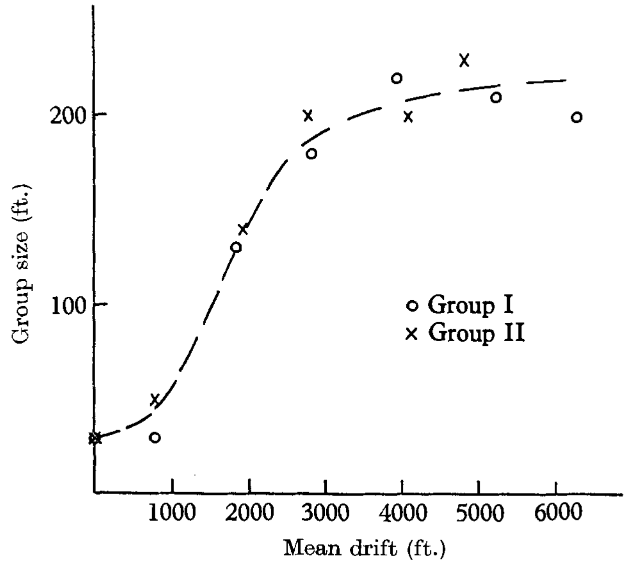

FIGURE 8. Typical case of 'regular' diffusion. Run no. 8, 22 June; dispersal of papers.

	Surface current		Diffusion constants		
Run no.	Direction (°)	Speed (cm/sec)	g_x	L (cm)	K_a (cm²/sec)
1	330	10	0·032	780	125
2	25	4	0·14	420	120 (before slick)
3	210	16	0·06	900	430 (before slick)
4	25	10	0·05	1200	300
5	215	8	0·25	1200	1200
6	240	15	0·03	500	115
7	230	18	0·03	2000	540
8	190	35	0·022	820	315

Note. Current direction is given on the same basis as wind direction in table 1, i.e. a current flowing from the north is 0°.

TABLE 2. Experimental results (regular diffusion)

drawn out into thin filaments and sheets, in accordance with some theoretical results of Batchelor (1952*b*). It is to be expected that the sharp concentration gradients set up in this manner will lead to accelerated molecular diffusion which contributes to the relative diffusion of the plume, as discussed by Saffman (1960).

General information on the test runs with dye is summarized in table 3. A typical cross-plume concentration profile obtained at the 60 cm level is shown

Exp. no.	Date	Wind direction (°)	Wind speed (m.p.h.)	Condition of lake	Duration of test run E.S.T.	Distance from shore at point of release (m)	Depth of water at point of release (m)	Rate of discharge of dye (g/sec)
1	11 Jul	200	8	2–3 ft. waves	13.00–16.00	800	15	0·2
2	15 Aug	200	10	3 ft. waves	8.00–14.00	400	7	0·2*
						800	15	0·15*
						1300	22	0·14*
3	20 Aug	200	12	2 ft. waves	8.00–14.00	800	15	0·2
4	3 Sep	270	3	6 in. swell	15.00–17.00	800	15	0·2

* Three plumes.

TABLE 3. General information on test runs (fluorometer)

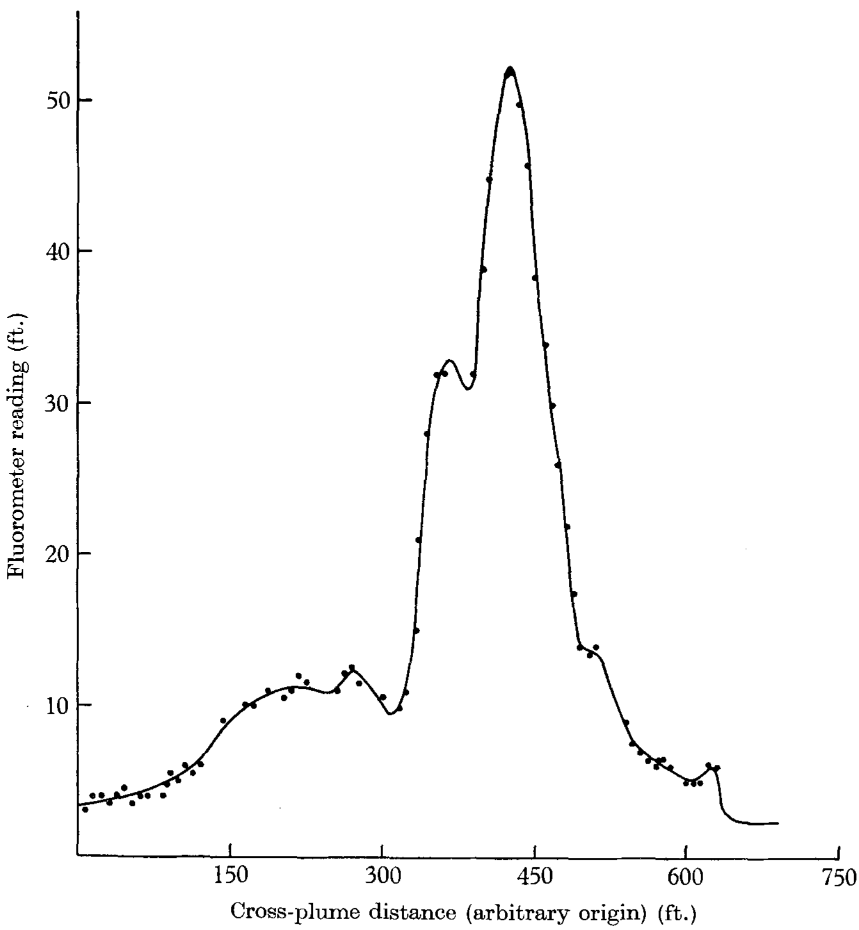

FIGURE 12. 'Typical' plume, 500 m from source. Run no. 22, 15 August.

in figure 12. The peak reading on the fluorometer corresponds to a dye concentration of about $1 \cdot 1 \times 10^{-7}$ g/c.c.

In order to eliminate to some extent random variations in the concentration profile 9 consecutive runs have been averaged and the result is shown as figure 13. Successive profiles were 'matched' by defining the plume centre as the point half-way between the edges. The number of profiles was not high enough to give a reliable statistical average, nor were the individual profiles uncorrelated, since the runs followed in fairly rapid succession. The average curve is, nevertheless, much more regular than any individual profile and leads one to suspect that if a large enough number of profiles were averaged the dye distribution would become Gaussian.

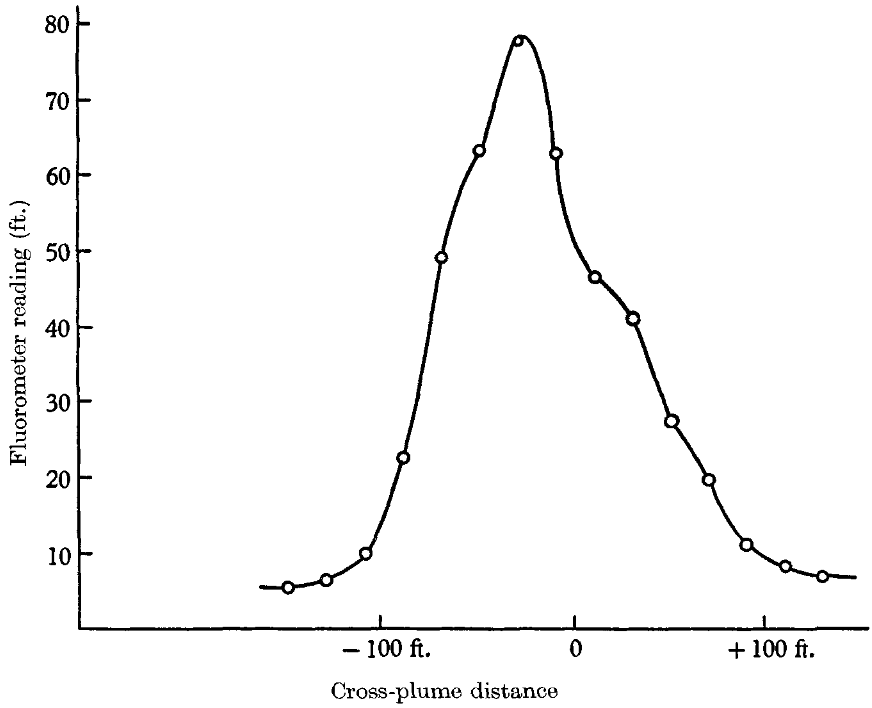

FIGURE 13. Average of nine consecutive runs at 820 m from the source. 20 August.

The lateral growth of a typical plume in a steady current is shown in figure 14. This was obtained on 11 July, when the entire experiment was devoted to a study of lateral growth and all samples were drawn at the 60 cm level. This curve shows a strong resemblance to the results obtained on the diffusion of floating objects. The three regimes of relative diffusion in a homogeneous field are clearly recognizable as pointed out before. On other occasions generally similar results were obtained, but with a different absolute magnitude of the diffusion constants. The diffusion constants for the various runs as far as they could be extracted from the data are summarized in table 4.

The growth of the plume is accompanied by a decrease of maximum concentration. Figure 15 shows this decrease with distance at the 60 cm level for the

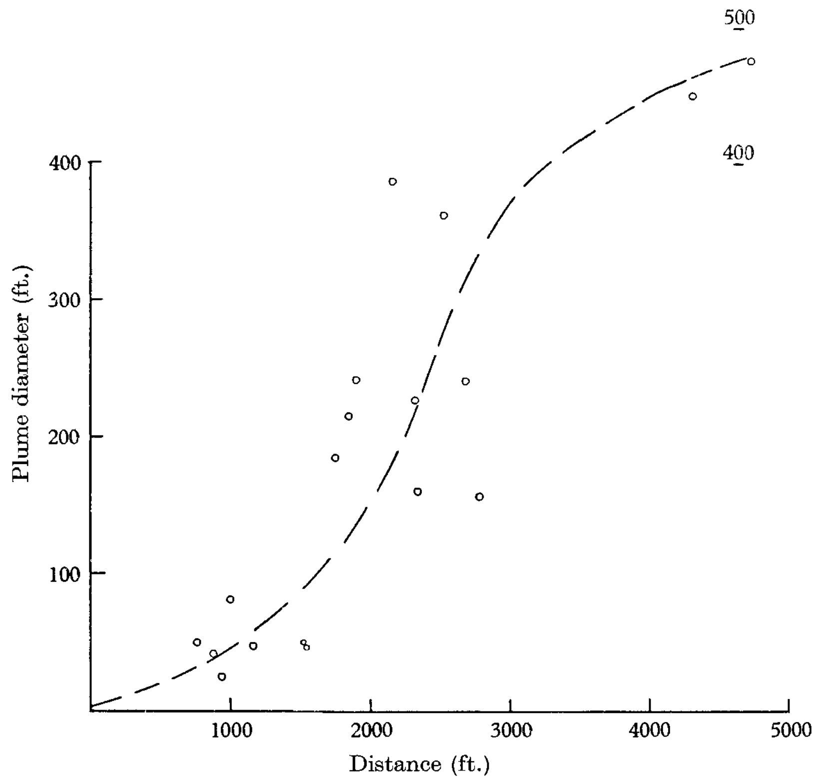

FIGURE 14. Lateral growth of plume. 11 July.

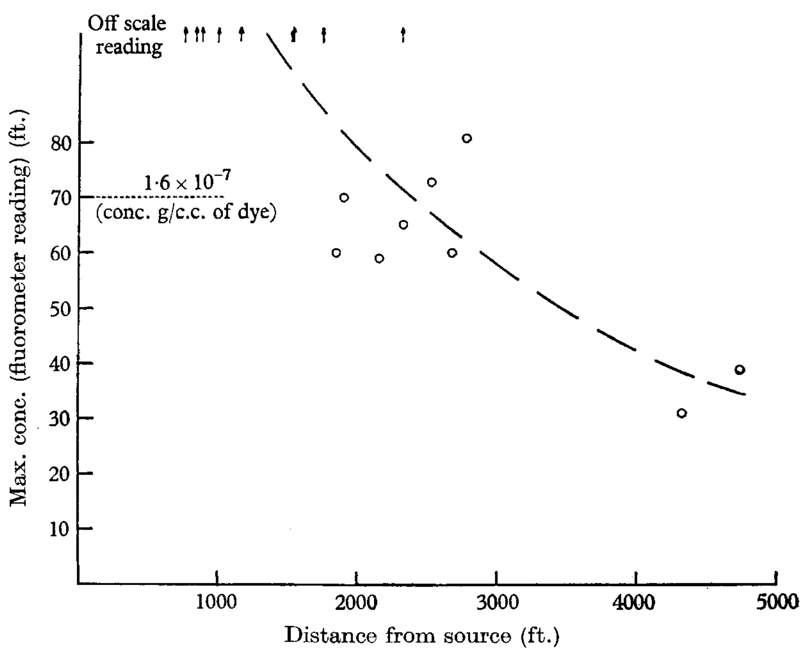

FIGURE 15. Decrease of maximum concentration at the 60 cm level. 11 July.

11 July run when a number of readings at different distances were taken. In order to have plentiful data on lateral growth an attempt was not made on that occasion to determine the maximum concentration close to the source. Nevertheless, the results show that at least beyond 600 m from the source the maximum concentration at the 60 cm level decreases approximately linearly with distance.

		Current		Diffusion constants		
Exp. no.	Date	Speed (cm/sec)	Direction (°)	Lateral gustiness g_y	Scale L (cm)	Diffusivity K_y (cm²/sec)
1	11 Jul	25	225	0·04	1070	500
2	15 Aug	7	250	—	—	300
3	20 Aug	18	210	—	—	400
4	3 Sep	5·5	270	—	—	470

TABLE 4. Diffusion constants

6. Complex patterns of diffusion

The considerable influence of a non-uniform velocity distribution on longitudinal diffusion rates was first pointed out by Taylor (1954) who calculated an 'effective' longitudinal diffusion coefficient K_x for diffusion in a circular pipe

$$K_x = 10{\cdot}1u^*a, \tag{6}$$

where u^* is the friction velocity and a is the radius of the pipe. In a similar analysis Elder (1959) has found the effective longitudinal diffusion coefficient in a broad, open channel

$$K_x = 5{\cdot}9u^*h, \tag{7}$$

where u^* is the friction velocity (formed with the shear stress at the bottom of the channel) and h is the depth of the channel. Ellison (1959) gave some corrections to Elder's theory, but these need not concern us here.

In the case of a wind-driven lake current a friction velocity may be formed from the shear stress on the free surface. According to Deacon (1962) the shear stress is very nearly

$$\tau = 0{\cdot}0012\rho_a U^2, \tag{8}$$

where ρ_a is the air density and U is the wind velocity at the 10 m level. The friction velocity in water is then

$$u^* = (\tau/\rho_w)^{\frac{1}{2}} = (0{\cdot}0012\rho_a/\rho_w)^{\frac{1}{2}}\, U, \tag{9}$$

where ρ_w is the water density.

At a typical wind speed of 5 m/sec this gives $u^* = 0{\cdot}5$ m/sec. The length scale corresponding to the depth of an open channel may be taken as the depth of the thermocline, which is ordinarily (in July and August) of the order of 10 m. Thus by equation (7) one may expect an effective longitudinal diffusivity of the order of $K_x = 3000\,\mathrm{cm^2/sec}$, which is considerably greater, although not quite by an order of magnitude, than the asymptotic lateral diffusivity K_y found by experiment. A very similar conclusion holds when the current is not wind driven, but is a gravity current following on a change of wind. Thus in steady, well-established currents one may expect longitudinal diffusion of dye to be much faster than lateral diffusion.

As regards diffusion in the vertical direction one strange effect is that the centre of gravity of the plume is likely to be displaced in the direction of lower transport velocities, i.e. from the surface downward. This may be inferred from an investigation of Smith (1957), figure 3. A second complicating factor is that vertical turbulent velocities must vanish at the free surface and therefore increase with depth. There must be a corresponding increase in the value of the vertical turbulent diffusivity, K_z. Dimensional arguments suggest the form:

$$K_z = mu^*z, \tag{10}$$

where m is a constant and z is depth measured from the free surface. The net effect on diffusion is that, as the cloud grows, the vertical spread is accelerated. A similar effect is also demonstrated by the analysis of Smith (1957).

Equations (2)–(6) and the accompanying qualitative arguments were all based on an Eulerian conception of diffusion and the results should hold for concentration measurements obtained with a fixed instrument. Instantaneous concentration profiles taken in a dye plume are likely to behave somewhat differently, particularly closer to the source where the effects of meandering are relatively strong. Thus there are likely to be random vertical (as well as horizontal) displacements of the plume's centre of gravity. There seem to be no theoretical results whatever to shed light on the problem of relative diffusion in shear flow.

Very scanty data only are available on the vertical spread of the dye plumes. Mostly these were obtained by dye-balance calculations supported by a few direct concentration measurements to a depth of up to 2·5 m. Effective vertical diffusivities varied between 0·1 and 1·5 cm^2/sec and were thus not far removed from the molecular constants which are of order 0·01 cm^2/sec. The rate of spread of the dye in the vertical was thus considerably less than in the horizontal. Typically, at a distance of 600 m from the source the horizontal diameter of the plume was 60 m, its vertical extension only about 1·5 m.

On one occasion, at a distance of 150 m from the source, definite evidence of meandering in the vertical has been found. Complicated patterns of dye distribution arose when, in a swinging current, the top layer changed direction more rapidly than those below.

An analysis of aerial photographs of diffusing dye patches (not plumes) taken in 1961 has yielded some interesting information on longitudinal diffusion rates. The patch of dye was released from a boat near the surface and during the initial hour or so the ratio of longitudinal to lateral diameter was around 2·5, slowly increasing. The diameters were scaled off directly from photographs. When the longitudinal diameter reached a length of about 150 m it began to grow very rapidly and the diameter ratio increased to about 10. After this the front edge of the patch became very diffuse so that the length could not be estimated with any confidence. However, on reaching a size of about 100 m the lateral diameter also began to grow very rapidly and the diameter ratio dropped again, possibly to around 5. Thus there is no doubt that the effective longitudinal diffusivity was considerably greater than the lateral one, but the simple picture based on effective diffusivity is complicated by the staggered appearance in the longitudinal and in the lateral direction of the phases of accelerated growth. Incidentally, the same phenomenon of staggered growth was also observed qualitatively in the diffusion of floating objects such as drift bottles and mimeograph paper. Numerically the value of 5 times lateral diffusivity for K_x gives a 'typical' value of about 2000 cm^2/sec in good agreement with the theoretical estimate.

7. Meandering

When a puff of smoke or dye is discharged into a turbulent fluid the subsequent dispersion may be viewed in a frame of reference fixed to the stationary source or else in a frame moving with the centre of gravity of the cloud. Batchelor (1949, 1952*a*) has tentatively used the description 'Eulerian' and 'Lagrangian' analysis, respectively, but it is now more usual to speak of 'absolute' and 'relative' dispersion. The connexion between the two is provided by the random motion of the centre of gravity of a diffusing cloud, to be called 'meandering'. Adopting a slightly different point of view one may regard 'absolute' diffusion (that which is measured by fixed-point instruments) as a superposition of the two component processes of 'relative' diffusion (which is measured by instruments moving with the diffusing group) and 'meandering', or bodily displacements of the entire group. In two recent papers Gifford (1959, 1960) has made use of this resolution in discussing, for example, concentration fluctuations measured by a fixed-point instrument.

It is evident from Gifford's work that the practical importance of meandering is very great, yet no experimental results on it seem to have been reported so far. In an attempt to fill this void, and also to provide a numerical basis for absolute diffusion estimates in Lake Huron, the meandering of dye plumes in the lake has been studied with the aid of aerial photography in the summer of 1962, with the kind co-operation of the R.C.A.F. Observing the lateral meandering of a continuous plume is clearly equivalent to observing the meandering of successive dye patches in the cross-flow direction and the plume provides a much easier target.

It may be shown theoretically (Csanady & Ellenton 1962) that the standard deviation of meandering in homogeneous turbulence increases approximately linearly at first (with time or distance drifted) and tends asymptotically to a constant value.

A typical composite photograph of a dye plume is shown in figure 16. The dye plume is seen to resemble a smoke plume, except perhaps for the relatively much slower rate of dispersion.

The influence of sampling time on the observed lateral standard deviation G_{22} of meandering is illustrated by figure 17. The number of readings in a given group is insufficient for a stable average, but the trend is nevertheless clear.

The most important information derived from this experiment is summarized

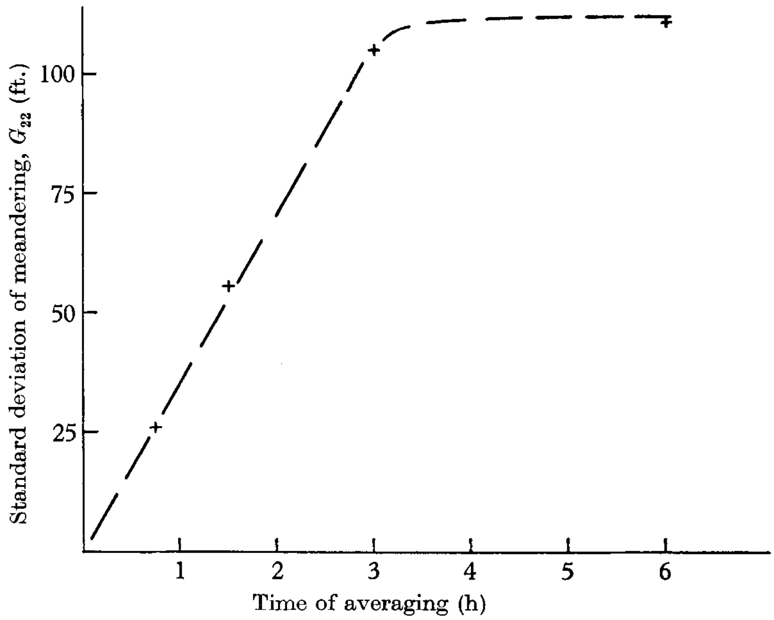

FIGURE 17. Growth of the standard deviation of meandering with sampling time, at 750 m distance from the source.

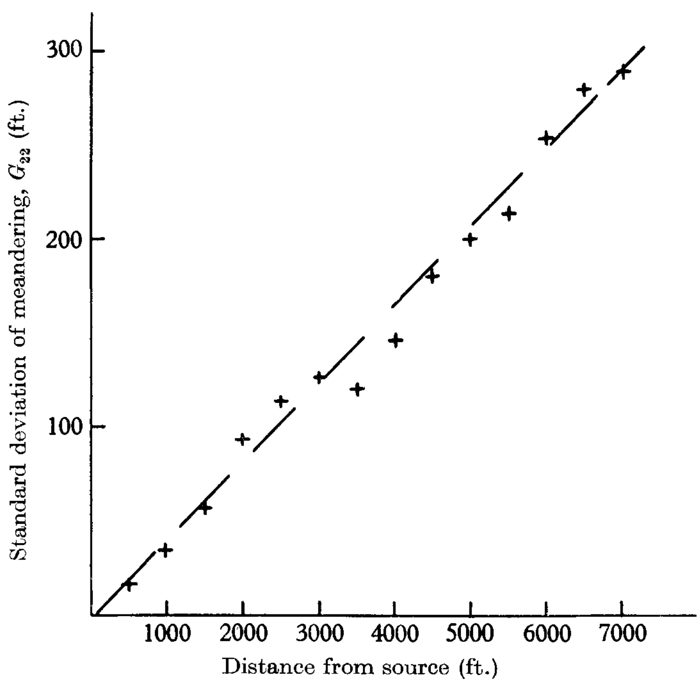

FIGURE 18. Growth of the standard deviation of meandering with distance from the source. Averaging time: 3–4 h.

in figure 18, regarding the behaviour of G_{22} with increasing distance from the source. The outstanding feature of the result is that even at 2100 m from the source there is no sign of G_{22} attaining an asymptotic value. Indeed, the growth of this quantity with distance is still nearly linear. As is found by relative diffusion measurements, the length scale of turbulence is of the order of 10 m and yet the standard deviation of meandering shows no signs of settling down to a value of 100 m. One can only conclude that some very large horizontal eddies were present. The 'wavelength' of the fluctuations in an individual plume's appearance (see again figure 16) was frequently of the order to 1000 m. Even if the lateral scale of such large eddies is smaller than their longitudinal scale (as would be the case in a boundary layer, for example) the asymptotic standard deviation of meandering could well be of comparable magnitude.

For practical applications we derive the important result that even at such distances meandering is a much more efficient dispersal mechanism in the lake than relative diffusion. Owing to poor relative diffusion a fixed-point instrument would show some occasional concentration peaks when swept by a plume, but a 4 or 5 h average reading would be quite low. Under these conditions the standard deviation of the absolute dispersion is not very far above the standard deviation of meandering and one may approach the problem in a first approximation by neglecting relative diffusion altogether.

The slowness of relative diffusion was well illustrated by a remark made by someone looking at the photograph in figure 16 for the first time, to the effect that it reminded him of the condensation trails of jet aircraft. Such 'contrails' are much narrower than smoke plumes because at high altitudes turbulence is next to non-existent. The situation in the lake is apparently not far different.

This investigation was undertaken by the Great Lakes Institute of the University of Toronto, under contract for the Atomic Energy of Canada Ltd. The project was under the direction of Dr R. E. Deane of the University of Toronto. The cooperation of Dr Deane and of Prof. H. K. Ellenton of the University of Waterloo in the experimental work is gratefully acknowledged. Thanks are due to Dr R. E. Munn, Meteorological Branch, Department of Transport, for access to some meteorological data and to Dr A. Cole of the Department of Health and Welfare for helpful suggestions.

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FIGURE 3. Irregular movement of three dye plumes.

FIGURE 2. Dye source in operation.

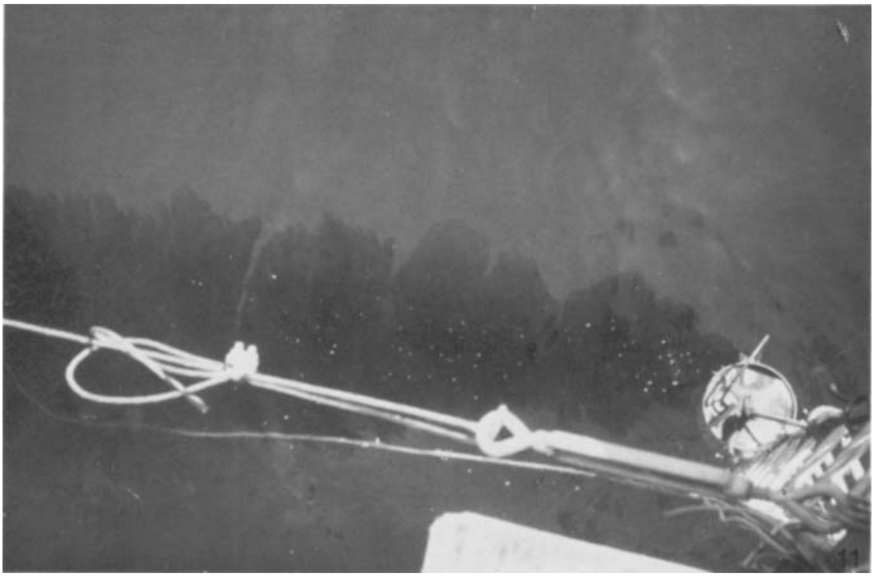

FIGURE 11. View of dye plume near source.

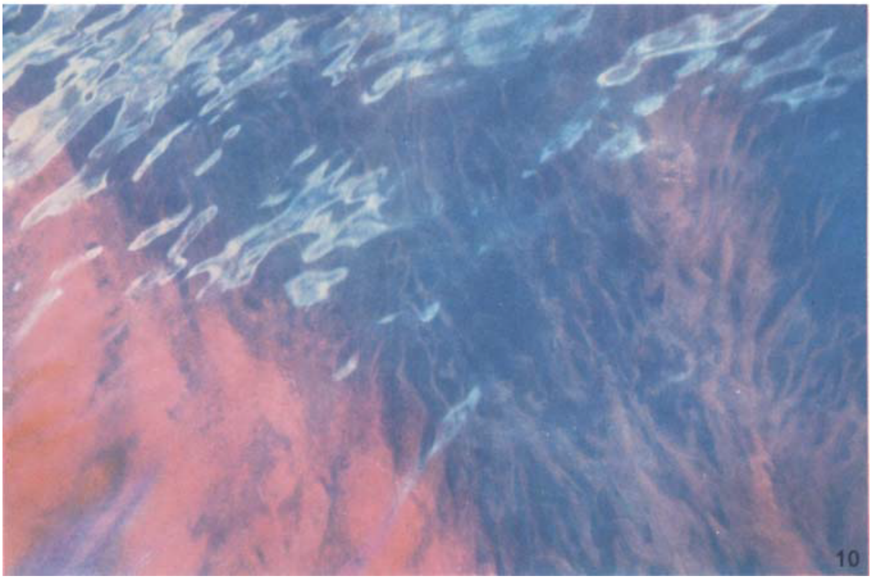

FIGURE 9. Irregular edge of diffusing dye plume.

FIGURE 10. Thin streaks of dye at the edge of a plume.

FIGURE 16. Composite photograph of dye plume.